Lab 11
Continuous Probability Distributions Name: _____

In exercises 1 - 3, the random variable x is normally distributed with mean $\mu = 174$ and standard deviation $\sigma = 20$. Find each probability.

1. Find $P(172 < X < 192)$

2. Find $P(X < 200)$

3. Find $P(X > 155)$

4. **Heights of Men** In a survey of U.S. men, the heights in the 20 –29 age group were normally distributed, with a mean of 69.4 inches and a standard deviation of 2.9 inches. Find the probability that a randomly selected study participant has a height that is (a) less than 66 inches, (b) between 66 and 72 inches, and (c) more than 72 inches, and (d) identify any unusual events. Explain your reasoning.

5. **Utility Bills** The monthly utility bills in a city are normally distributed, with a mean of \$100 and a standard deviation of \$12. Find the probability that a randomly selected utility bill is (a) less than \$70, (b) between \$90 and \$120, and (c) more than \$140.

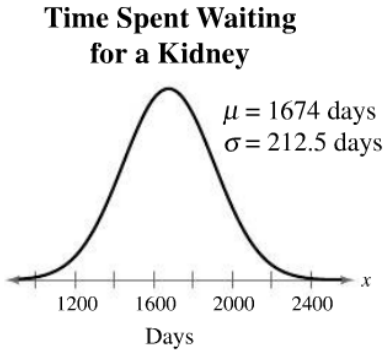
6. The average monthly mortgage payment including principal and interest is $982 in the United States. If the standard deviation is approximately $180 and the mortgage payments are approximately normally distributed, find the probability that a randomly selected monthly payment is (a) at least $1050, (b) over $900, and (c) at most $550

7. Data from the paper "Fetal Growth Parameters and Birth Weight: Their Relationship to Neonatal Body Composition" (Ultrasound in Obstetrics and Gynecology [2009]: 441–446) suggest that a normal distribution with mean of 3,500 grams and standard deviation of 600 grams is a reasonable model for the probability distribution of birth weights of full-term babies. What proportion of birth weights are between 2,900 and 4,700 grams?

8. What proportion of birth weights are higher than 4800 grams?

9. What birth weight represents the 90th percentile?

10. **Heights of Women** In a survey of women in the United States (ages 20 –29), the mean height was 64.2 inches with a standard deviation of 2.9 inches. Assume that heights are normally distributed. (Adapted from National Center for Health Statistics)
    (a) What height represents the 95th percentile?
    (b) What height represents the first quartile?



11. **Kidney Transplant Waiting Times** The time spent (in days) waiting for a kidney transplant for people ages 35– 49 can be approximated by a normal distribution, as shown in the figure. (Adapted from Organ Procurement and Transplantation Network)
    (a) What waiting time represents the 80th percentile?
    (b) What waiting time represents the first quartile?

12. **Bananas** The annual per capita consumption of fresh bananas (in pounds) in the United States can be approximated by a normal distribution, with a mean of 10.4 pounds and a standard deviation of 3 pounds. (Adapted from U.S. Department of Agriculture)
    (a) What is the smallest annual per capita consumption of bananas that can be in the top 10% of consumptions?
    (b) What is the largest annual per capita consumption of bananas that can be in the bottom 5% of consumptions?

13. The amount of time spent by a statistical consultant with a client at their first meeting is a random variable that has a normal distribution with a mean value of 60 minutes and a standard deviation of 10 minutes.
    a) What is the probability that more than is spent at the first meeting?
    b) What amount of time is exceeded by only 10% of all clients at a first meeting?

14. Find the cumulative area that corresponds to a z score of -1.51.

15. Find the cumulative area that corresponds to a z score of 2.37.

16. Find the probability $P(z < 1.35)$.

17. Find the probability $P(z \leq 1.35)$.

18. Find the probability $P(z < -0.18)$.

19. Find the probability $P(z > 2.358)$.

20. Find the probability $P(-1.2 < z < 0.18)$.

21. Find the probability $P(z \geq 3.5)$.

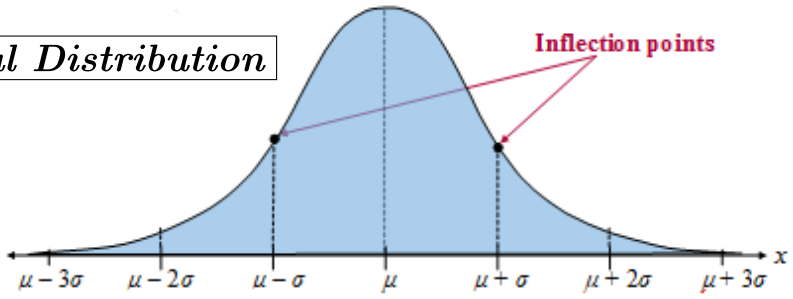
22. Find the z score associated with P_{90}, the 90th percentile.

23. Find the z score associated with quartile 3.

24. Find the z score associated with the 5th percentile.

In this chapter, we study the most important continuous probability distribution in statistics — ***the normal distribution***. Normal distributions are bell-shaped curves that model many sets of measurements in nature, industry and business. For instance, height, weights and blood pressures of humans, lengths of pregnancies, the lifetimes of washers and dryers and even housing costs are all normally distributed.

The Normal Distribution

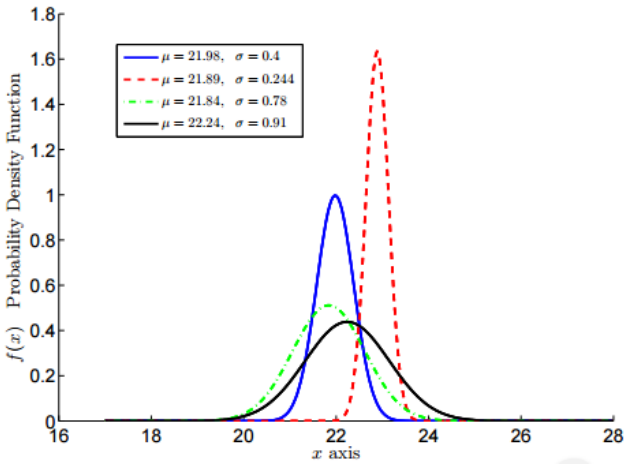


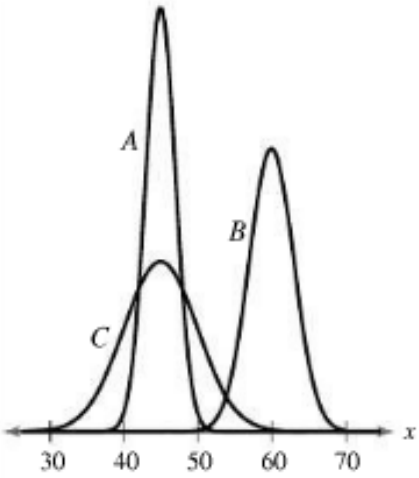
Properties of a Normal Distn.

1. The graph of a normal distribution is called the ***normal curve***.
2. A normal curve is bell-shaped and can be graphed using the function formula

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty \leq x \leq \infty$$

3. The mean, $x = \mu$, locates the center of the distribution. The vertical line, $x = \mu$ is an axis of symmetry for the PDF.
4. The total area under the curve is equal to one.
5. The normal curve approaches, but never touches the x-axis as it extends farther and farther away from the mean.
6. Between $\mu - \sigma$ and $\mu + \sigma$ (in the center of the curve), the graph curves downward. The graph curves upward to the left of $\sigma - \mu$ and to the right of $\sigma + \mu$. The points at which the curve changes from curving upward to curving downward are called the ***inflection points***.
7. The population standard deviation, σ, describes how spread out over the x-axis the PDF curve is. Large values of σ decrease the height of the peak and increase the spread of the distribution (along the x axis; small values of σ raise the height of the peak and decrease the spread.





Use the normal curves in the figure (left) to answer the following questions.

1. Which normal curve has the greatest mean?

2. Which normal curve has the greatest standard deviation?

3. Which normal curve has the least standard deviation?

4. What equation describes the line (axis) of symmetry for curve C?

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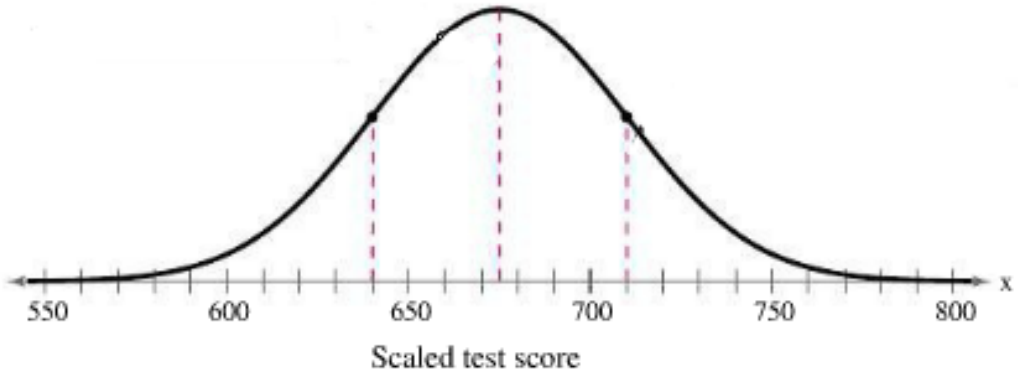
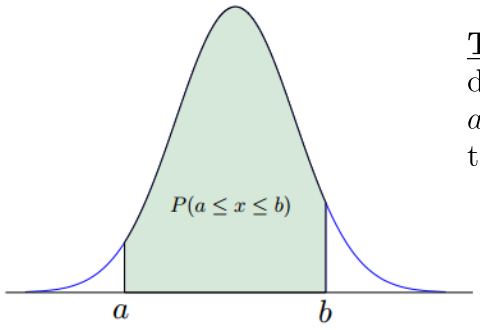
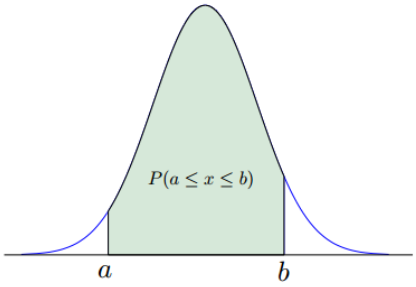
5. The scaled test scores for a New York State Grade 8 Math Test are normally distributed. The normal curve shown below represents this distribution. What is the mean test score? Estimate a value for the standard deviation of this normal distribution.





Theorem: The probability that a continuous random variable x assumes a value in the interval from a to b is the area under the probability density function between vertical lines $x = a$ and $x = b$.

6. Use your calculator's "normalCDF" command to find the probability that a randomly selected New York State Grade 8 Math Test score is between and including 600 and 650.



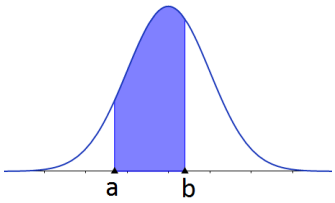
Theorem: The probability that a continuous random variable x assumes a value in the interval from a to b is the area under the probability density function between vertical lines $x = a$ and $x = b$.

Guidelines: How to find area under a normal curve that has mean, μ, and standard deviation, σ. Use the calculator's "normalCdf" command.

To access the normalCdf command press [2nd] [VARS] [2]

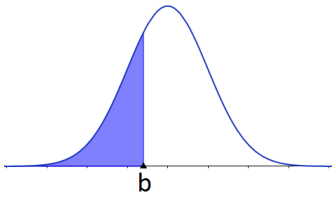
1) For $P(a \leq x \leq b)$ or $P(a < x < b)$

use $normalCdf(a, b, \mu, \sigma)$



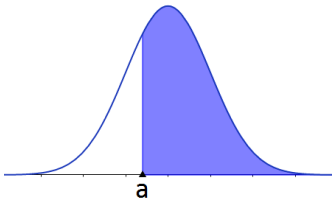
2) For $P(x \leq b)$ or $P(x < b)$

use $normalCdf(-10^9, b, \mu, \sigma)$



3) For $P(x \geq a)$ or $P(x > a)$

use $normalCdf(a, 10^9, \mu, \sigma)$



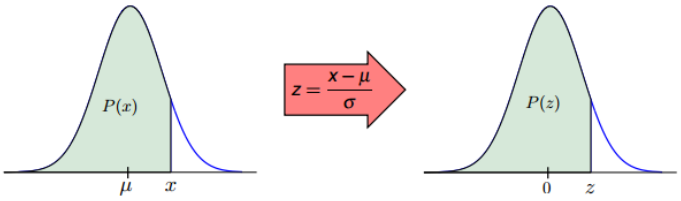
Example

7. A Honda Y2J diesel engine is used to run each packaging assembly line at the Lay's Potato Chip Factory. Let x represent the lifespan of a Honda Y2J diesel engine. Accepted values for the mean value and standard deviation of x are 25 years and 2 years, respectively. Suppose that the probability distribution of x is approximately normal. What is the probability that the lifespan of a randomly selected Honda Y2J diesel engine will be
   (a) not more than 23 years
   (b) at least 23 years
   (c) between 23 and 25 years
   (d) answer part a using the z-table

Definition: The phrase **"cumulative area"** is used to indicate an area under a curve that is LEFT of a specific number located on the horizontal axis.

Before we had calculators, statisticians had to find areas (probabilities) using calculus routines; or they used a "standardized table" (also called a "z-table") that lists several ***cumulative areas*** (probabilities) for one normal curve, called the ***Standard Normal Distribution***.

Definition: The ***Standard Normal Distribution*** is the normal distribution that has a mean, μ equal to 0, and a standard deviation, σ equal to 1.



Properties of the Standard Normal Distn.

- The Standard Normal Distribution is a collection of z scores.
- The Standard Normal Curve is centered at the origin.
- The mean, $\mu = 0$ and standard deviation, $\sigma = 1$.
- The cumulative area is close to 0 for z-scores close to $z = -3.49$.
- The cumulative area increases as the z-scores increase.
- The cumulative area for z = 0 is 0.5000 or 50%
- The cumulative area is close to 1 (or 100%) for z-scores close to z = 3.49.

How did statisticians use the Standard Normal Distribution and a "z-table" to find areas (probabilities) under *non-standard* normal curves?

The answer is that each data value of the normally distributed random variable x was transformed into a z score (using the z-score formula). This standardization process converts any non-standard normal curve whose mean is not 0 or whose standard deviation is not 1 to the **Standard Normal Distribution**.

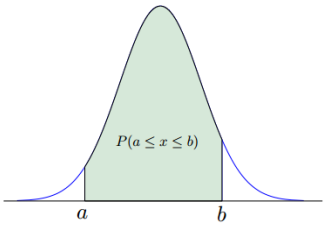


Figure 1: A Non-Standard Normal Distribution

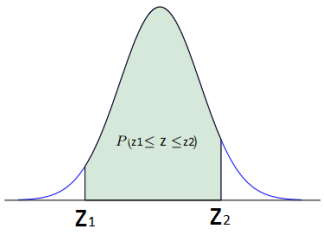


Figure 2: The Standard Normal Distribution

After this transformation takes place, **the area under the nonstandard normal curve that falls in the interval from 'a' to 'b' is the same as that under the standard normal curve within the corresponding z-scores of 'a' and 'b'**. The area between the two z-boundaries is then found in the z-table.

NEGATIVE *z* Scores

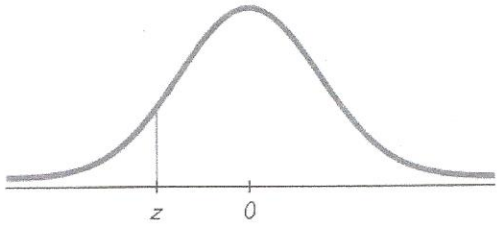


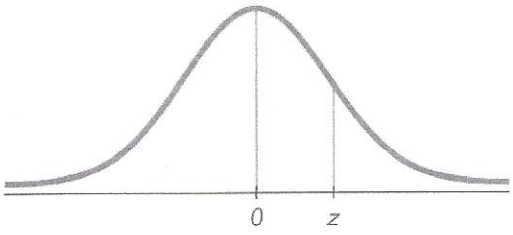
TABLE A-2 Standard Normal (z) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	* .0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	* .0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

NOTE: For values of z below −3.49, use 0.0001 for the area.

*Use these common values that result from interpolation:

z score	Area
−1.645	0.0500
−2.575	0.0050



POSITIVE *z* Scores

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	* .9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	* .9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.50 and up	.9999									

NOTE: For values of *z* above 3.49, use 0.9999 for the area.

*Use these common values that result from interpolation:

z score	Area
1.645	0.9500
2.575	0.9950

Common Critical Values

Confidence Level	Critical Value
0.90	1.645
0.95	1.96
0.99	2.575